

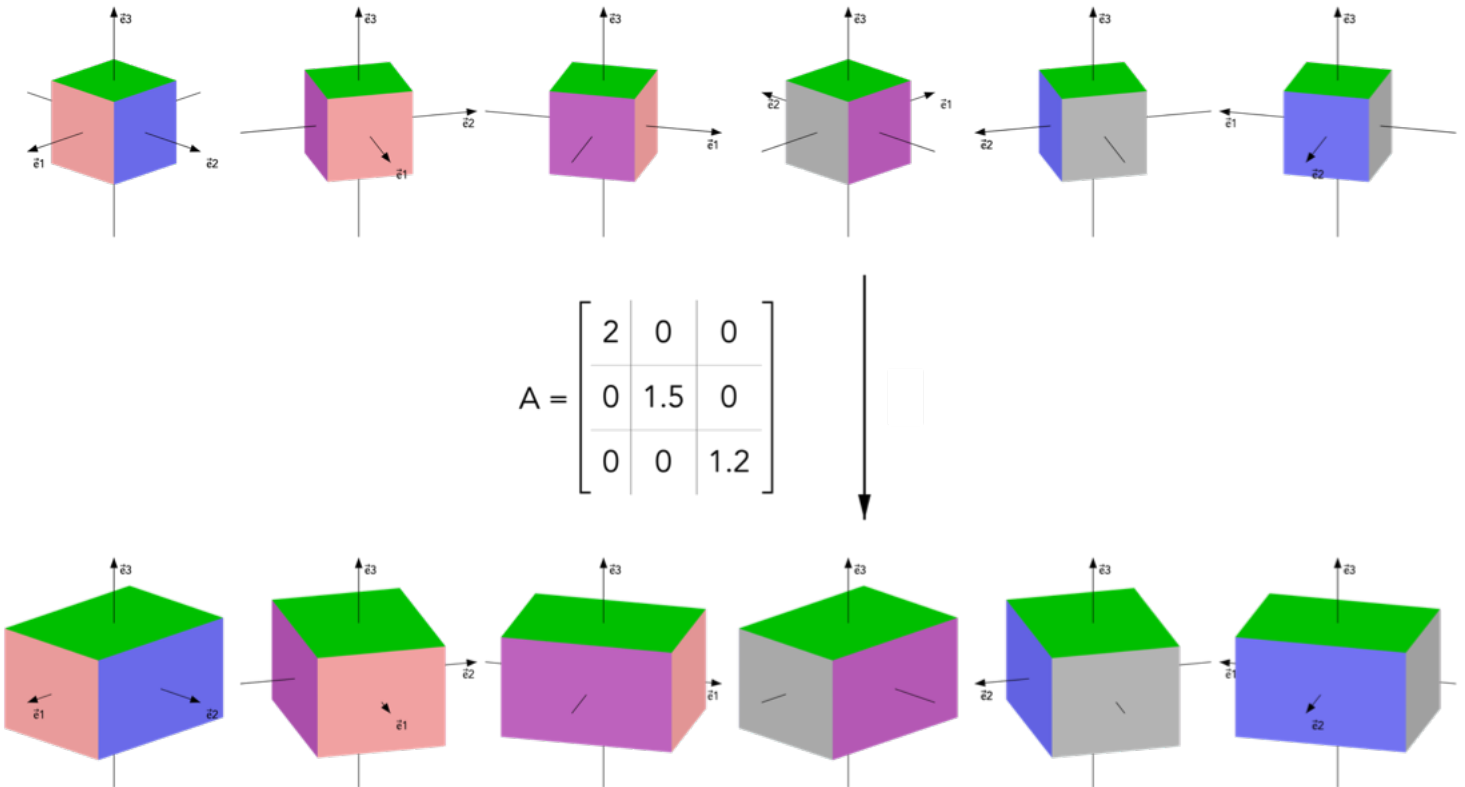
Scaling

has the following form:

$$\begin{bmatrix} s_1 & 0 & 0 \\ 0 & s_2 & 0 \\ 0 & 0 & s_3 \end{bmatrix}$$

$$\begin{bmatrix} s_1 & 0 & 0 \\ 0 & s_2 & 0 \\ 0 & 0 & s_3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} s_1 x_1 \\ s_2 x_2 \\ s_3 x_3 \end{bmatrix}$$

$$x'_1 = s_1 x_1, \quad x'_2 = s_2 x_2, \quad x'_3 = s_3 x_3$$



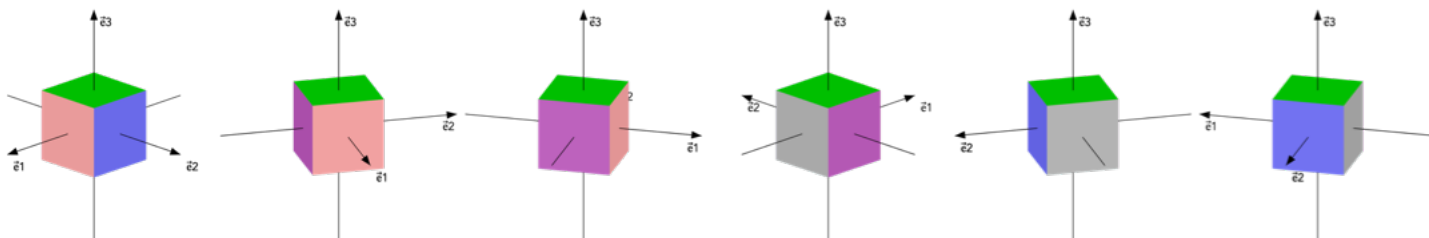


Elementary shear of x_1 by x_2

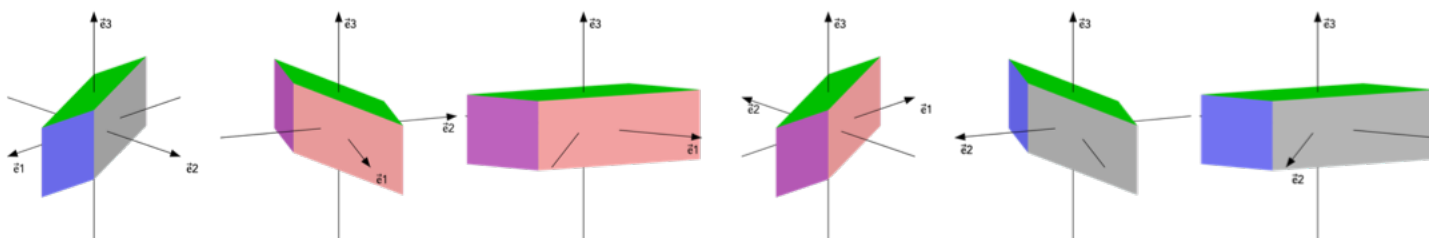
has the following form:
$$\left[\begin{array}{c|c|c} 1 & s & 0 \\ \hline 0 & 1 & 0 \\ \hline 0 & 0 & 1 \end{array} \right]$$

$$\left[\begin{array}{c|c|c} 1 & s & 0 \\ \hline 0 & 1 & 0 \\ \hline 0 & 0 & 1 \end{array} \right] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} x_1 + s x_2 \\ x_2 \\ x_3 \end{bmatrix}$$

$$x_1' = x_1 + s x_2, \quad x_2' = x_2, \quad x_3' = x_3$$



$$A = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



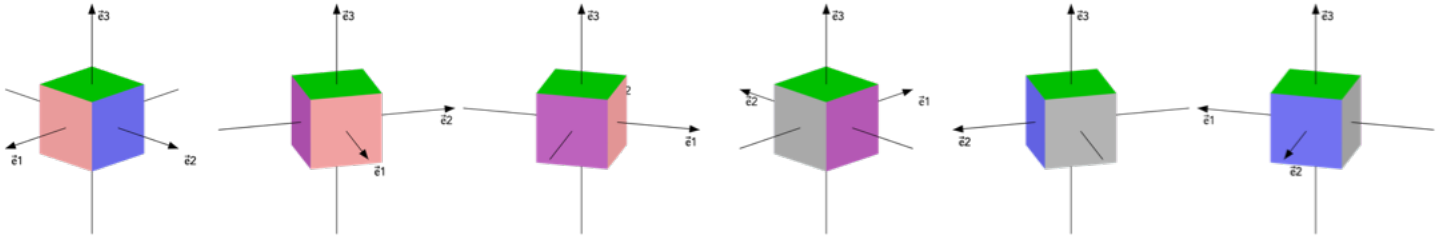
Elementary shear of x_1 by x_3

has the following form:

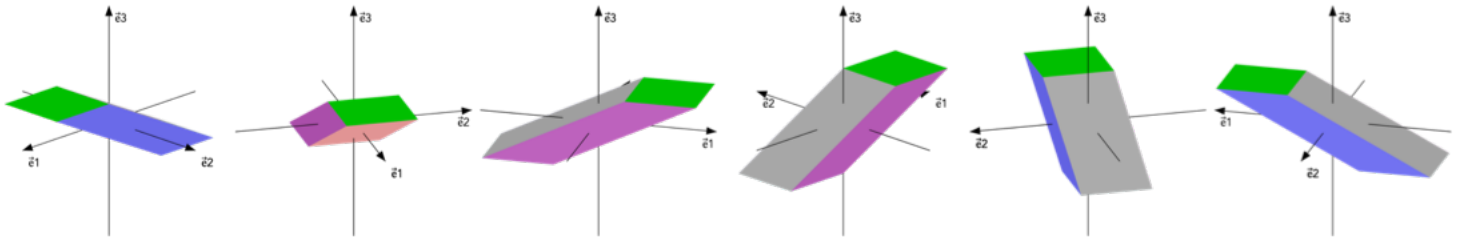
$$\begin{bmatrix} 1 & 0 & s \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & s \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} x_1 + s x_3 \\ x_2 \\ x_3 \end{bmatrix}$$

$$x_1' = x_1 + s x_3, \quad x_2' = x_2, \quad x_3' = x_3$$



$$A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



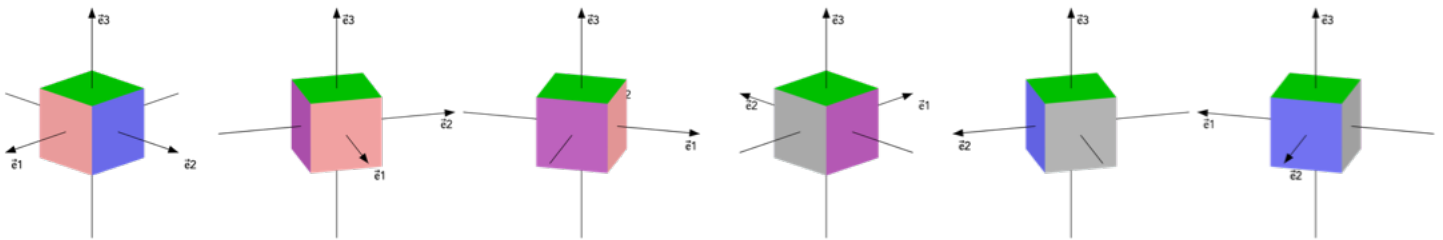
Elementary shear of x_2 by x_1

has the following form:

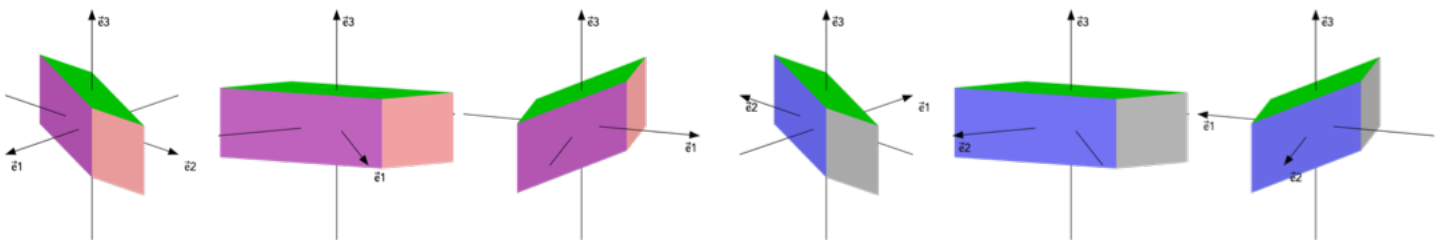
$$\begin{bmatrix} 1 & 0 & 0 \\ s & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ s & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} x_1 \\ s x_1 + x_2 \\ x_3 \end{bmatrix}$$

$$x_1' = x_1, \quad x_2' = x_2 + s x_1, \quad x_3' = x_3$$



$$A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

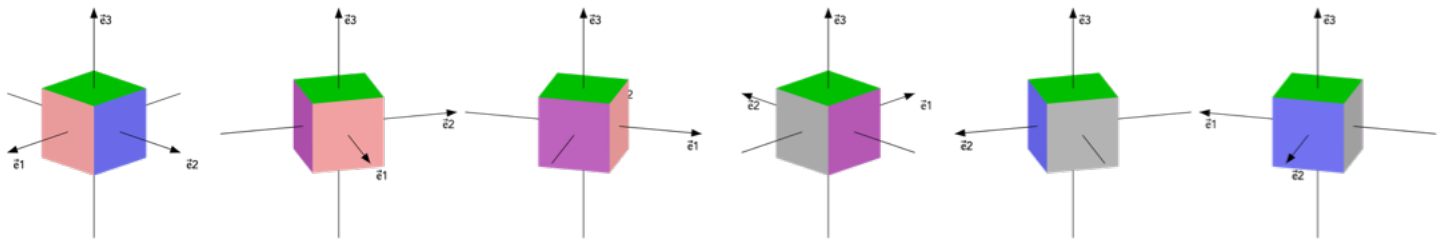


Elementary shear of x_2 by x_3

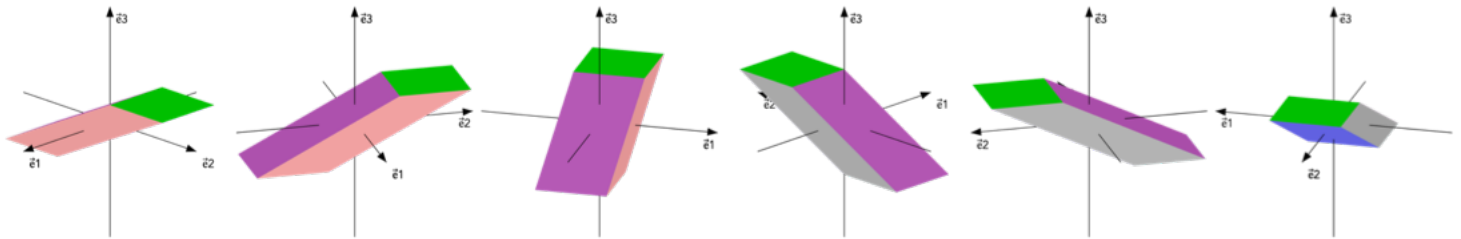
has the following form:
$$\left[\begin{array}{c|c|c} 1 & 0 & 0 \\ \hline 0 & 1 & s \\ \hline 0 & 0 & 1 \end{array} \right]$$

$$\left[\begin{array}{c|c|c} 1 & 0 & 0 \\ \hline 0 & 1 & s \\ \hline 0 & 0 & 1 \end{array} \right] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 + s x_3 \\ x_3 \end{bmatrix}$$

$$x_1' = x_1, \quad x_2' = x_2 + s x_3, \quad x_3' = x_3$$



$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$



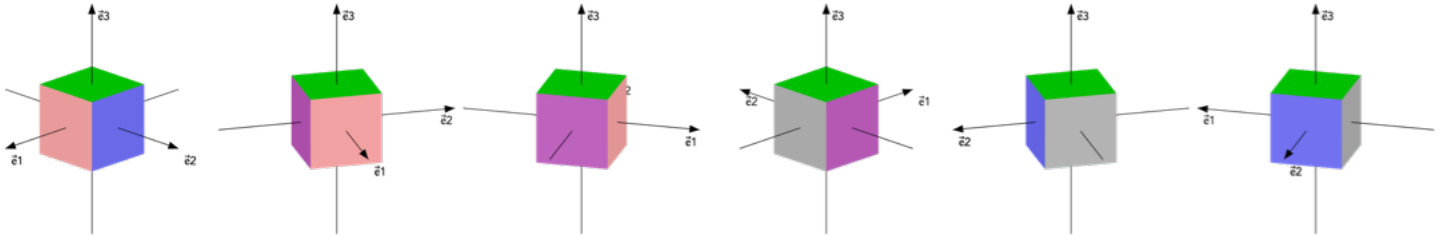
Elementary shear of x_3 by x_1

has the following form:

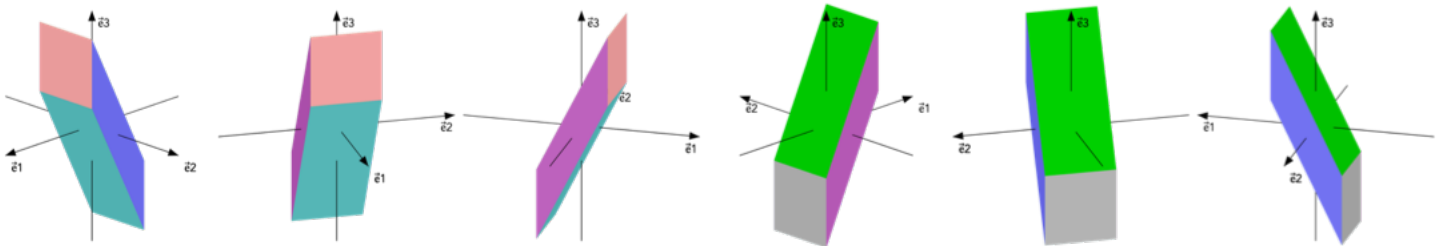
$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ s & 0 & 1 \end{bmatrix}$$

$$\left[\begin{array}{c|c|c} 1 & 0 & 0 \\ \hline 0 & 1 & 0 \\ \hline s & 0 & 1 \end{array} \right] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ s x_1 + x_3 \end{bmatrix}$$

$$x_1' = x_1, \quad x_2' = x_2, \quad x_3' = x_3 + s x_1$$



$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix}$$



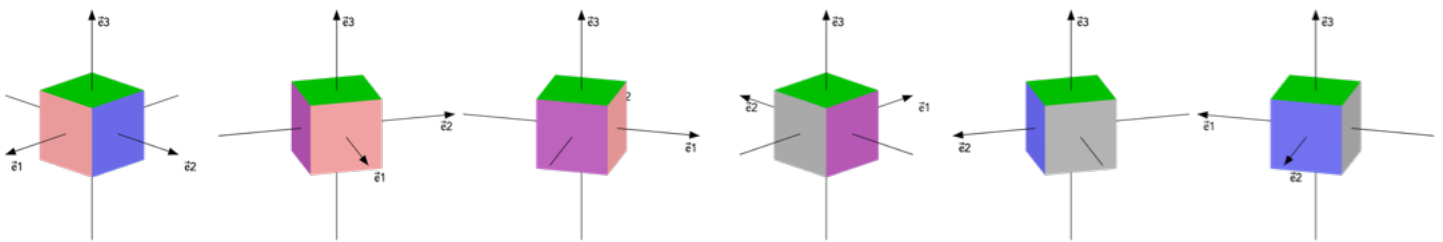
Elementary shear of x_3 by x_2

has the following form:

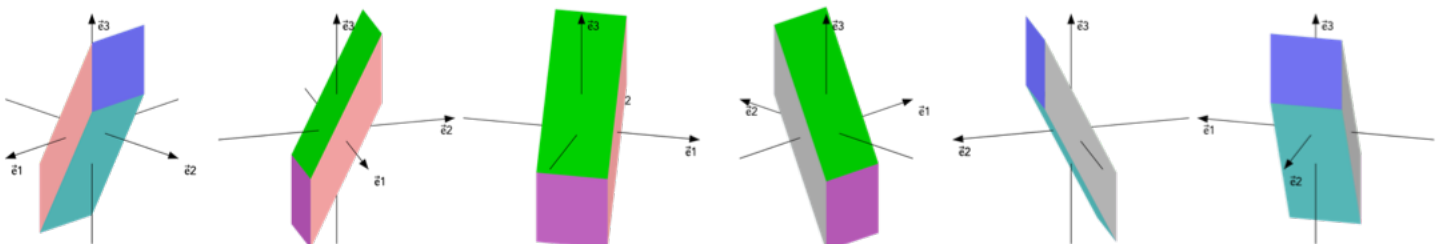
$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & & & \\ \hline 0 & 1 & 0 & & & \\ \hline 0 & s & 1 & & & \end{array} \right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & & & \\ \hline 0 & 1 & 0 & & & \\ \hline 0 & s & 1 & & & \end{array} \right] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ s x_2 + x_3 \end{bmatrix}$$

$$x_1' = x_1, \quad x_2' = x_2, \quad x_3' = x_3 + s x_2$$



$$A = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & & & \\ \hline 0 & 1 & 0 & & & \\ \hline 0 & 2 & 1 & & & \end{array} \right]$$

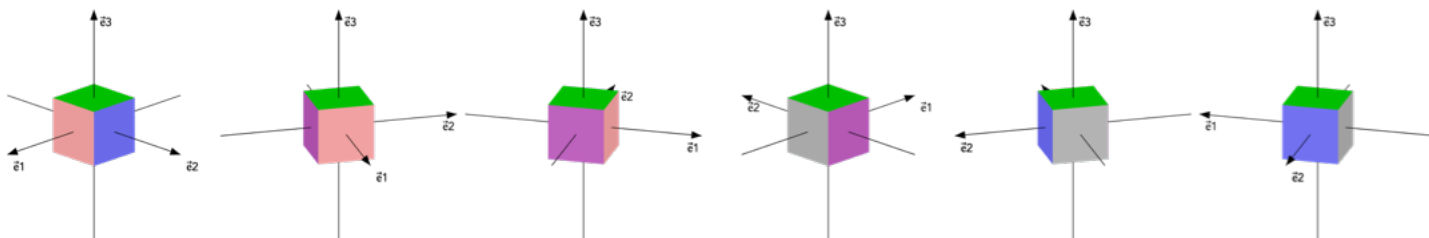


Shear x_3 by x_1 and x_2

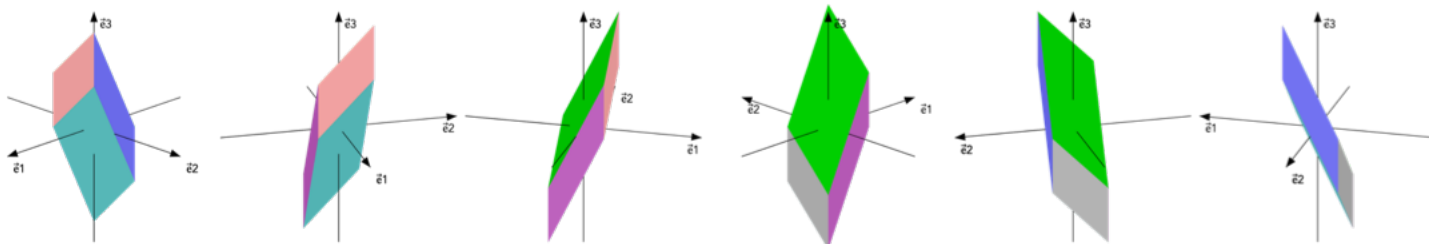
has the following form:
$$\left[\begin{array}{c|c|c} 1 & 0 & 0 \\ \hline 0 & 1 & 0 \\ \hline s_1 & s_2 & 1 \end{array} \right]$$

$$\left[\begin{array}{c|c|c} 1 & 0 & 0 \\ \hline 0 & 1 & 0 \\ \hline s_1 & s_2 & 1 \end{array} \right] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ s_1 x_1 + s_2 x_2 + x_3 \end{bmatrix}$$

$$x_1' = x_1, \quad x_2' = x_2, \quad x_3' = x_3 + s_1 x_1 + s_2 x_2$$



$$A = \left[\begin{array}{c|c|c} 1 & 0 & 0 \\ \hline 0 & 1 & 0 \\ \hline 2 & 1 & 1 \end{array} \right]$$



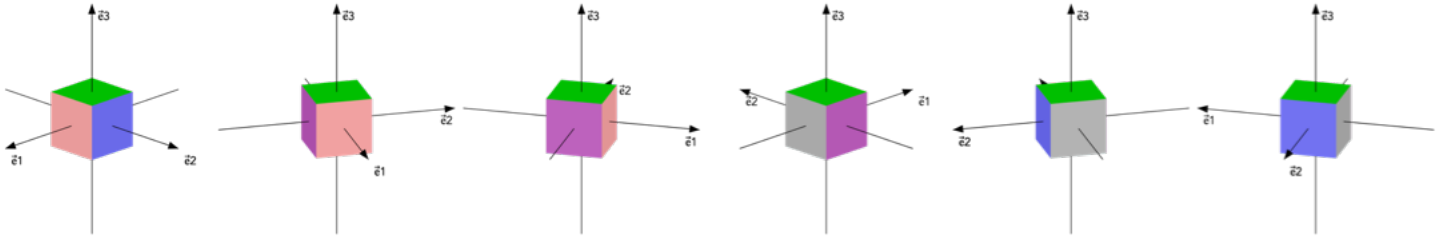
Shear x_2 by x_1 and x_3

has the following form:

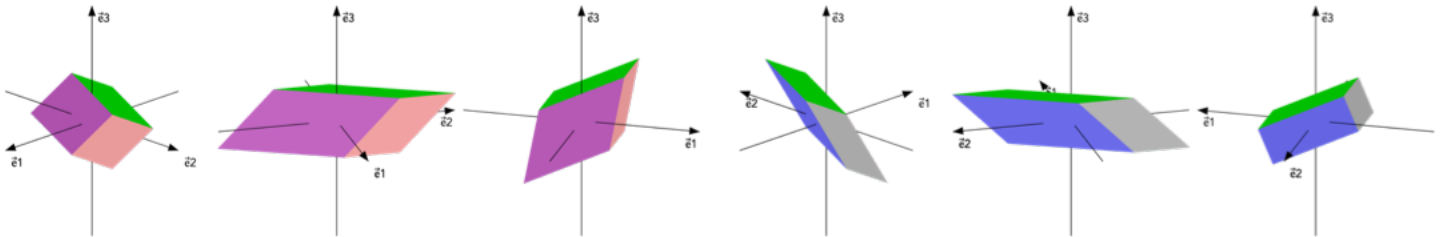
$$\left[\begin{array}{c|c|c} 1 & 0 & 0 \\ \hline s_1 & 1 & s_2 \\ \hline 0 & 0 & 1 \end{array} \right]$$

$$\begin{bmatrix} 1 & 0 & 0 \\ s_1 & 1 & s_2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} x_1 \\ s_1 x_1 + x_2 + s_2 x_3 \\ x_3 \end{bmatrix}$$

$$x'_1 = x_1, \quad x'_2 = x_2 + s_1 x_1 + s_2 x_3, \quad x'_3 = x_3$$



$$A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$



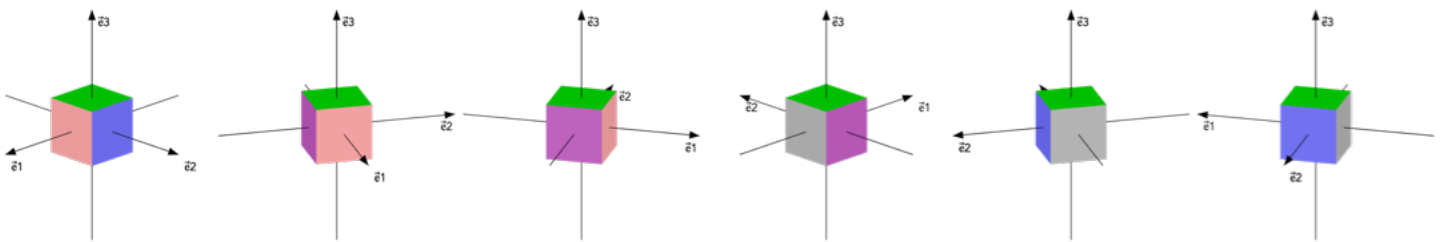
Shear x_1 by x_2 and x_3

has the following form:

$$\begin{bmatrix} 1 & s_1 & s_2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & s_1 & s_2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} x_1 + s_1 x_2 + s_2 x_3 \\ x_2 \\ x_3 \end{bmatrix}$$

$$x_1' = x_1 + s_1 x_2 + s_2 x_3, \quad x_2' = x_2, \quad x_3' = x_3$$



$$A = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

