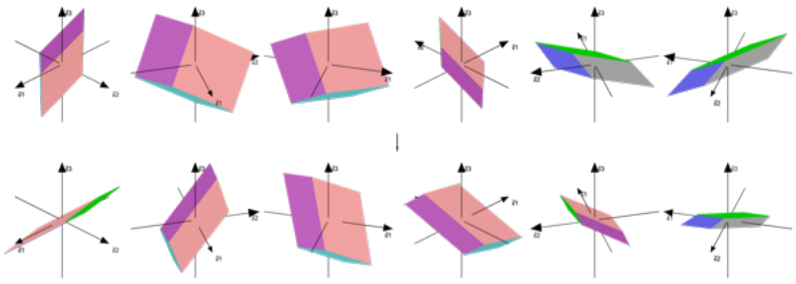
Step 1 of 8

Make M[2,1] = 0 using pivot M[1,1]

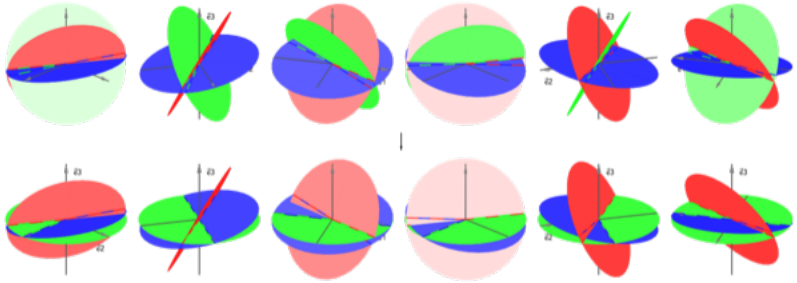
$$R2 \leftarrow R2 - \left(\frac{M[2,1]}{M[1,1]}\right) \times R1$$

$$\begin{bmatrix} 3 & 5 & -3 \\ 3 & 5 & 3 \\ 0 & -1 & 5 \end{bmatrix} \rightarrow \begin{bmatrix} 3 & 5 & -3 \\ 0 & 0 & 6 \\ 0 & -1 & 5 \end{bmatrix}$$

Column directions
Shearing space:
- all columns move together
- sheared space straightens along $\vec{e}2$ direction



Row normal planes (red: r1, green: r2, blue: r3)
coefficient of x_1 is eliminated in row 2
↓
green plane adjusts so that
(plane normal) · $\vec{e}1 = 0$



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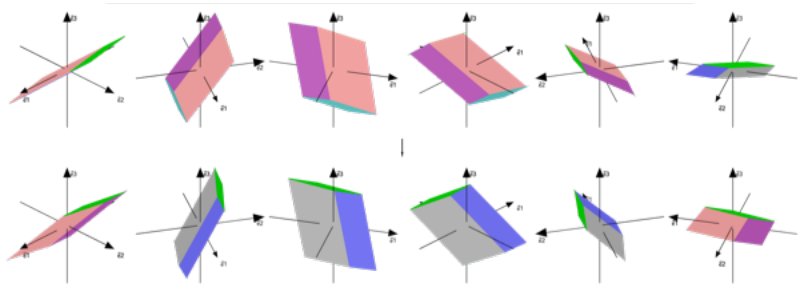
Step 2 of 8

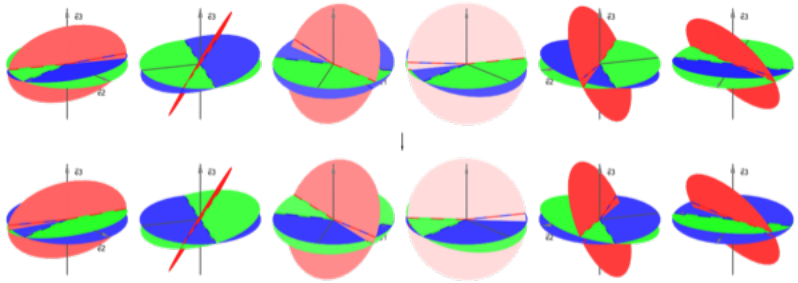
Swap pivot into position: R3 ⇔ R2

$$\begin{bmatrix} 3 & 5 & -3 \\ 0 & 0 & 6 \\ 0 & -1 & 5 \end{bmatrix} \rightarrow \begin{bmatrix} 3 & 5 & -3 \\ 0 & -1 & 5 \\ 0 & 0 & 6 \end{bmatrix}$$

Column directions
Swapping rows is equivalent to reflection across
- plane $x_3 = x_2$
- plane normal: $\vec{e}2 - \vec{e}3$



Row normal planes (red: r1, green: r2, blue: r3)
blue and green planes exchange places



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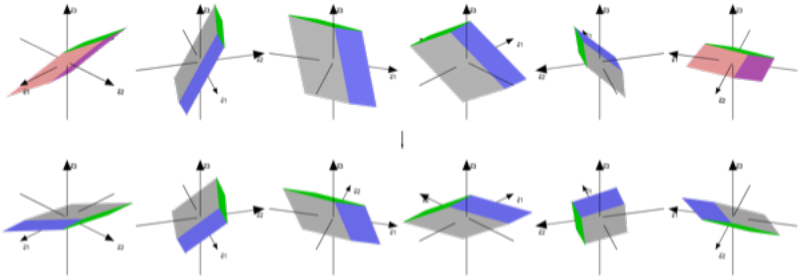
Step 3 of 8
Make pivot M[3,3] = 1

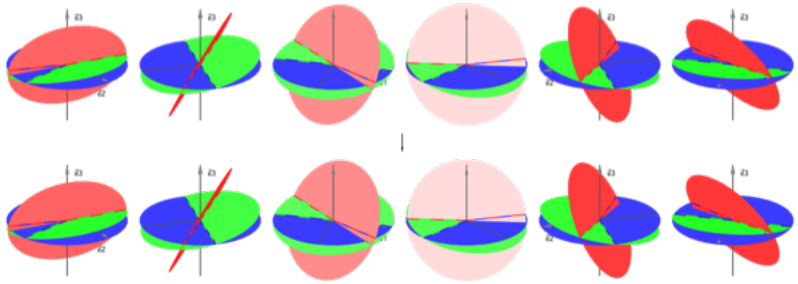
$$R3 \leftarrow \left(\frac{1}{M[3,3]}\right) \times R3$$

$$\left[\begin{array}{ccc|c} 3 & 5 & -3 & \\ 0 & -1 & 5 & \\ 0 & 0 & 6 & \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 3 & 5 & -3 & \\ 0 & -1 & 5 & \\ 0 & 0 & 1 & \end{array}\right]$$

Column directions
Scaling space:
all columns scale
together along $\vec{e}3$ direction by ≈0.167
scaling coefficient > 0
↓
directions are preserved



Row normal planes (red: r1, green: r2, blue: r3)
planes remain unchanged



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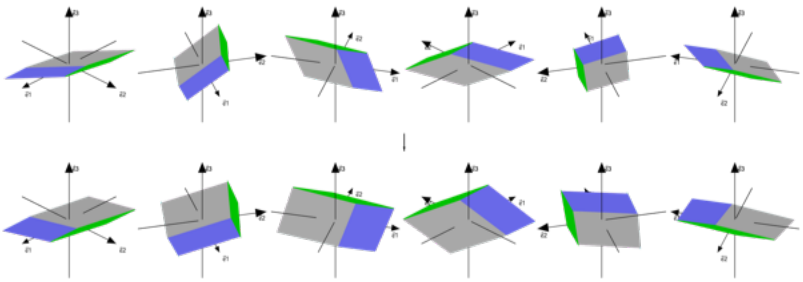
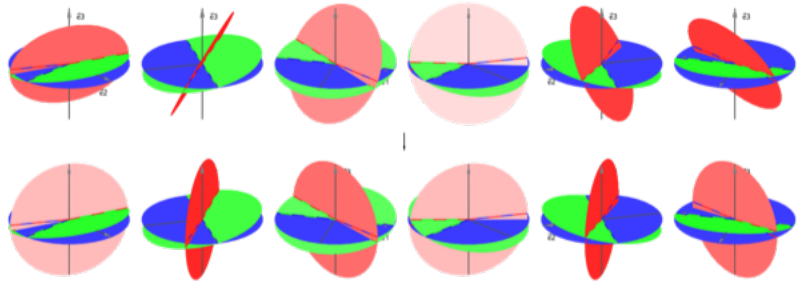
Step 4 of 8
Make M[1,3] = 0 using pivot M[3,3]
R1 ← R1 - M[1,3]*R3

$$\begin{bmatrix} 3 & 5 & -3 \\ 0 & -1 & 5 \\ 0 & 0 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 3 & 5 & 0 \\ 0 & -1 & 5 \\ 0 & 0 & 1 \end{bmatrix}$$

Column directions
Shearing space:
• all columns move together
• sheared space straightens along $\vec{e}1$ direction



Row normal planes (red: r1, green: r2, blue: r3)
coefficient of x_3 is eliminated in row 1
↓
red plane adjusts so that
(plane normal) · $\vec{e}3$ = 0



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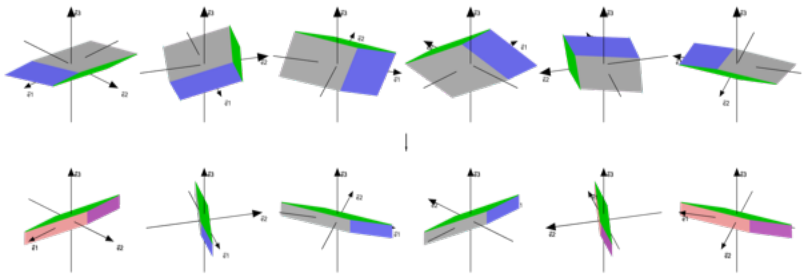
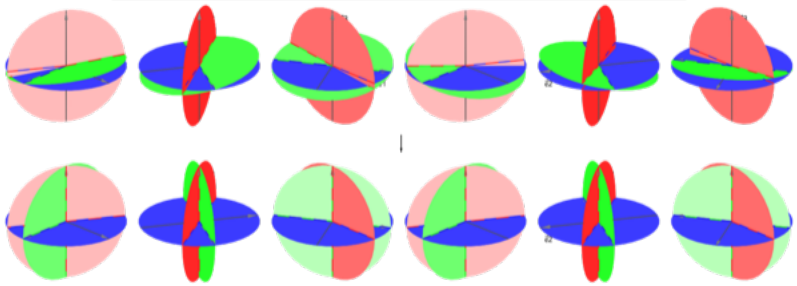
Step 5 of 8
Make M[2,3] = 0 using pivot M[3,3]
R2 ← R2 - M[2,3]*R3

$$\begin{bmatrix} 3 & 5 & 0 \\ 0 & -1 & 5 \\ 0 & 0 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 3 & 5 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Column directions
Shearing space:
• all columns move together
• sheared space straightens along $\vec{e}2$ direction



Row normal planes (red: r1, green: r2, blue: r3)
coefficient of x_3 is eliminated in row 2
↓
green plane adjusts so that
(plane normal) · $\vec{e}3$ = 0



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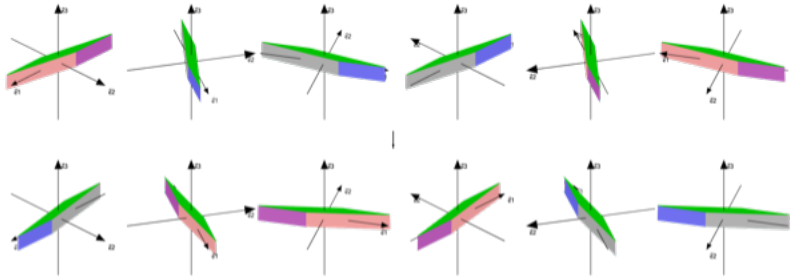
Step 6 of 8
Make pivot M[2,2] = 1

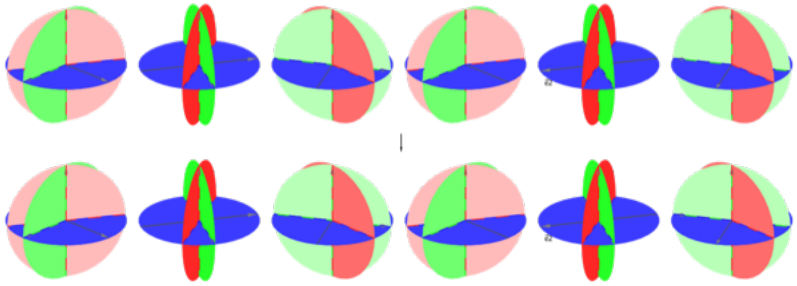
$$R2 \leftarrow \left(\frac{1}{M[2,2]}\right) \times R2$$

$$\left[\begin{array}{ccc|c} 3 & 5 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 3 & 5 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array}\right]$$

Column directions
Scaling space:
all columns scale
together along $\vec{e}2$ direction by -1
scaling coefficient < 0
↓
direction along this axis is reversed



Row normal planes (red: r1, green: r2, blue: r3)
planes remain unchanged



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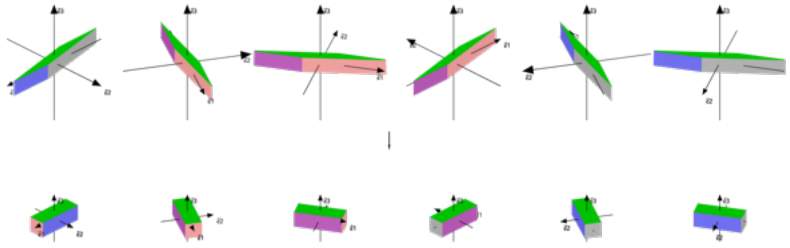
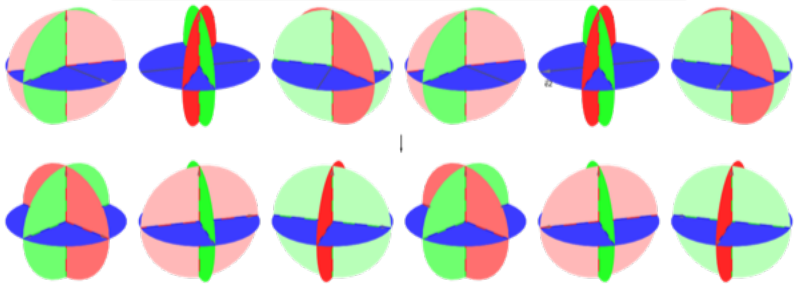
Step 7 of 8
Make M[1,2] = 0 using pivot M[2,2]
R1 ← R1 - M[1,2]*R2

$$\begin{bmatrix} 3 & 5 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 3 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Column directions
Shearing space:
• all columns move together
• sheared space straightens along $\vec{e}1$ direction



Row normal planes (red: r1, green: r2, blue: r3)
coefficient of x_2 is eliminated in row 1
↓
red plane adjusts so that
(plane normal) · $\vec{e}2$ = 0



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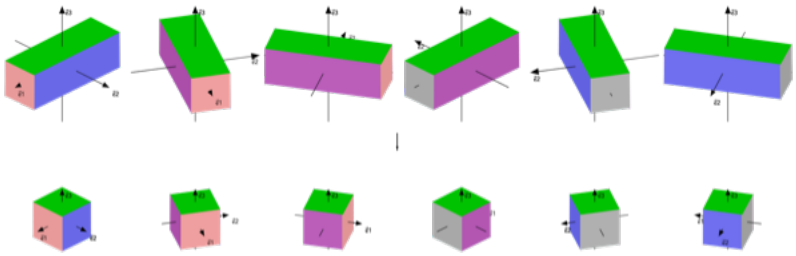
Step 8 of 8
Make pivot M[1,1] = 1

$$R1 \leftarrow \left(\frac{1}{M[1,1]}\right) \times R1$$

$$\left[\begin{array}{ccc} 3 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array}\right] \rightarrow \left[\begin{array}{ccc} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array}\right]$$

Column directions
Scaling space:
all columns scale
together along $\vec{e}1$ direction by ≈0.333
scaling coefficient > 0
↓
directions are preserved



Row normal planes (red: r1, green: r2, blue: r3)
planes remain unchanged