Step 1 of 7

Make M[2,1] = 0 using pivot M[1,1]

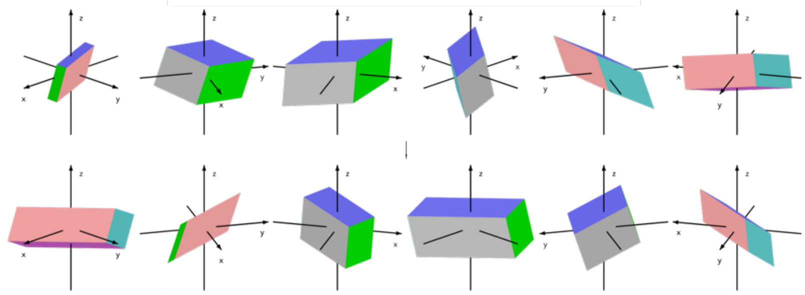
$$R2 \leftarrow R2 - \left(\frac{M[2,1]}{M[1,1]}\right) \times R1$$

$$\begin{bmatrix} 2 & 0 & 5 \\ 3 & 1 & 2 \\ 1 & 3 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 2 & 0 & 5 \\ 0 & 1 & -5.5 \\ 1 & 3 & 0 \end{bmatrix}$$

Column space fills $\mathbb{R}^3$

Shearing space:

- all columns move together
- sheared space straightens along $\vec{e}_2$ direction



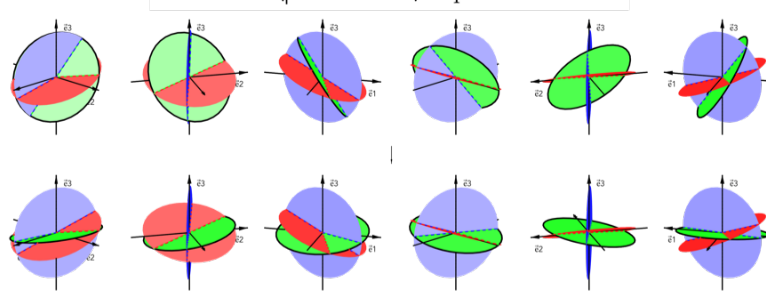
Row normal planes (red: r1, green: r2, blue: r3)

coefficient of x_1 is eliminated in row 2

↓

green plane adjusts so that

(plane normal) · $\vec{e}_1$ = 0



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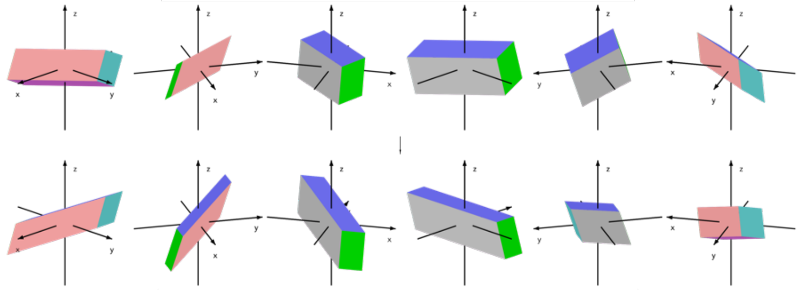
Step 2 of 7

Make M[3,1] = 0 using pivot M[1,1]

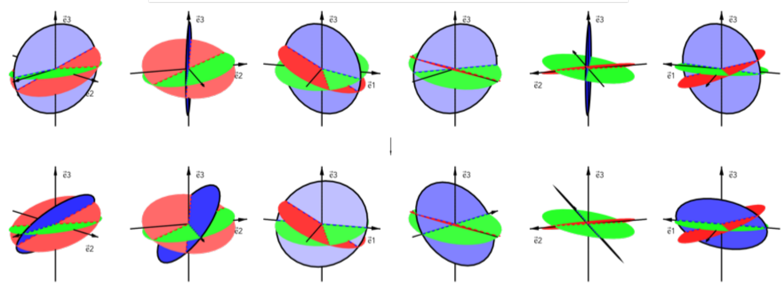
$$R3 \leftarrow R3 - \left(\frac{M[3,1]}{M[1,1]}\right) \times R1$$

$$\begin{bmatrix} 2 & 0 & 5 \\ 0 & 1 & -5.5 \\ 1 & 3 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 2 & 0 & 5 \\ 0 & 1 & -5.5 \\ 0 & 3 & -2.5 \end{bmatrix}$$

Column space fills $\mathbb{R}^3$
Shearing space:
• all columns move together
• sheared space straightens along $\vec{e}_3$ direction



Row normal planes (red: r1, green: r2, blue: r3)
coefficient of x_1 is eliminated in row 3
↓
blue plane adjusts so that
(plane normal) · $\vec{e}_1 = 0$



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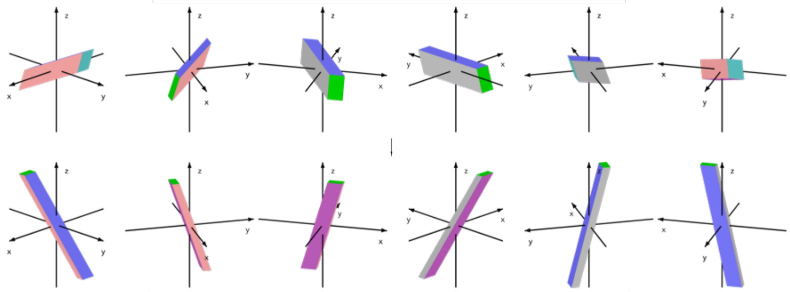
Step 3 of 7
Make M[3,2] = 0 using pivot M[2,2]

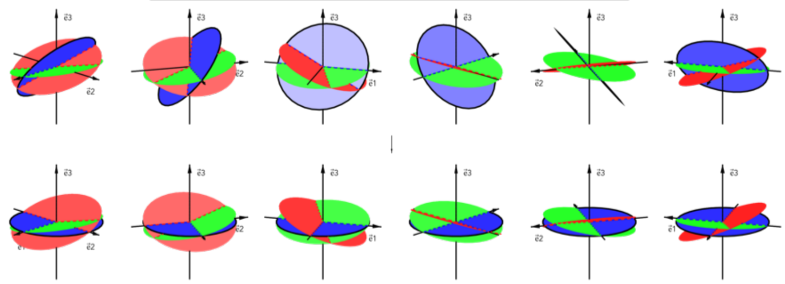
$$R3 \leftarrow R3 - \left(\frac{M[3,2]}{M[2,2]}\right) \times R2$$

$$\begin{bmatrix} 2 & 0 & 5 \\ 0 & 1 & -5.5 \\ 0 & 3 & -2.5 \end{bmatrix} \rightarrow \begin{bmatrix} 2 & 0 & 5 \\ 0 & 1 & -5.5 \\ 0 & 0 & 14 \end{bmatrix}$$

Column space fills $\mathbb{R}^3$
Shearing space:
- all columns move together
- sheared space straightens along $\vec{e}_3$ direction



Row normal planes (red: r1, green: r2, blue: r3)
coefficient of x_2 is eliminated in row 3
↓
blue plane adjusts so that
(plane normal) · $\vec{e}_2 = 0$



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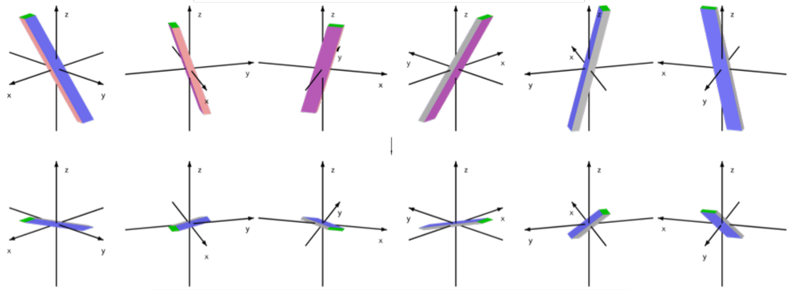
Step 4 of 7
Make pivot M[3,3] = 1

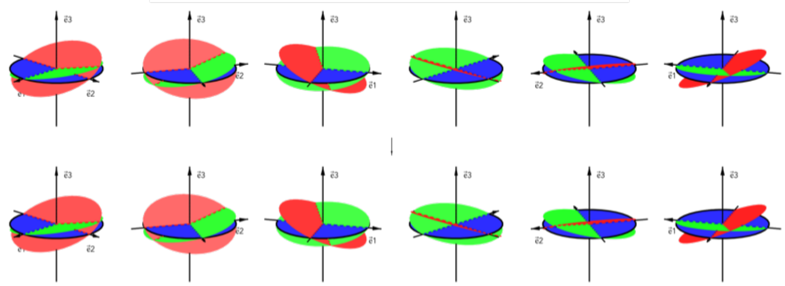
$$R3 \leftarrow \left(\frac{1}{M[3,3]}\right) \times R3$$

$$\begin{bmatrix} 2 & 0 & 5 \\ 0 & 1 & -5.5 \\ 0 & 0 & 14 \end{bmatrix} \rightarrow \begin{bmatrix} 2 & 0 & 5 \\ 0 & 1 & -5.5 \\ 0 & 0 & 1 \end{bmatrix}$$

Column space fills $\mathbb{R}^3$
Scaling space:
all columns scale
together along $\vec{e}_3$ direction by ≈0.071
scaling coefficient > 0
↓
directions are preserved



Row normal planes (red: r1, green: r2, blue: r3)
scaled plane remains unchanged



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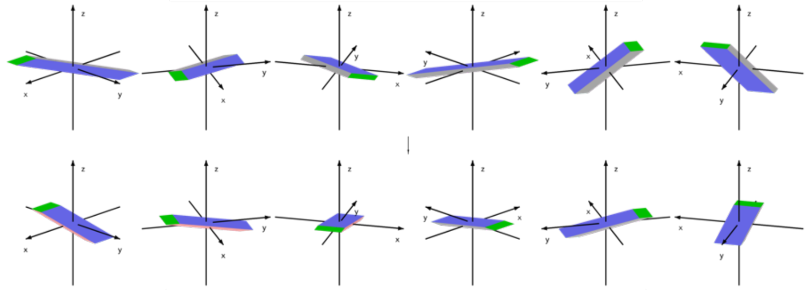
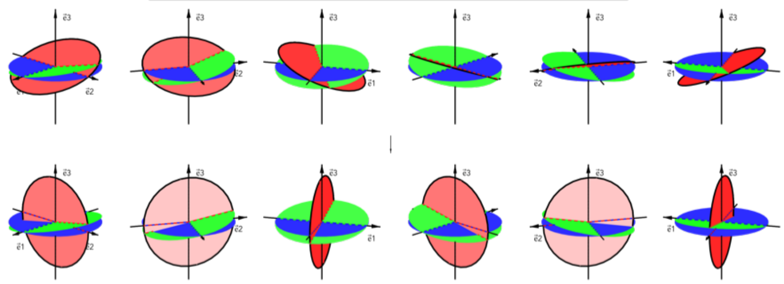
Step 5 of 7
Make M[1,3] = 0 using pivot M[3,3]
R1 ← R1 - M[1,3]*R3

$$\begin{bmatrix} 2 & 0 & 5 \\ 0 & 1 & -5.5 \\ 0 & 0 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & -5.5 \\ 0 & 0 & 1 \end{bmatrix}$$

Column space fills $\mathbb{R}^3$
Shearing space:
- all columns move together
- sheared space straightens along $\vec{e}_1$ direction



Row normal planes (red: r1, green: r2, blue: r3)
coefficient of x_3 is eliminated in row 1
↓
red plane adjusts so that
(plane normal) $\cdot\ \vec{e}_3 = 0$



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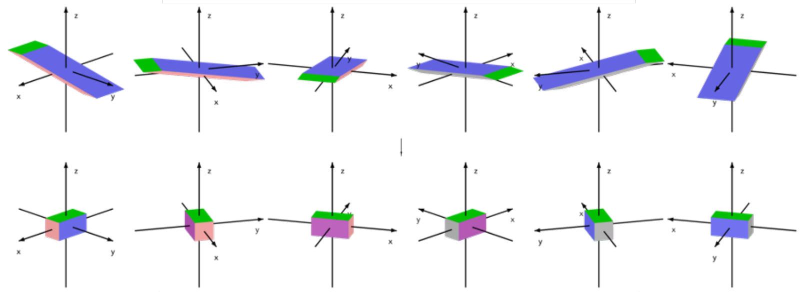
Step 6 of 7
Make M[2,3] = 0 using pivot M[3,3]
R2 ← R2 - M[2,3]*R3

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & -5.5 \\ 0 & 0 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Column space fills $\mathbb{R}^3$

Shearing space:

- all columns move together
- sheared space straightens along $\vec{e}_2$ direction



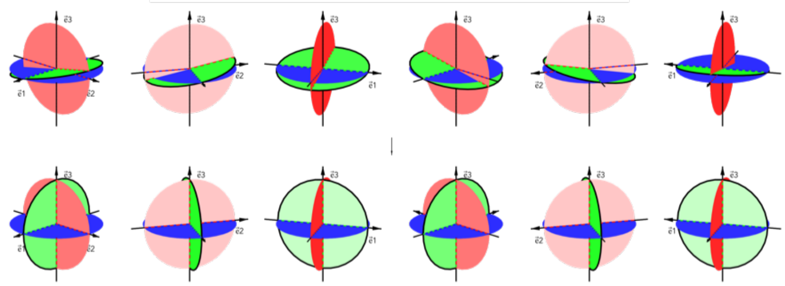
Row normal planes (red: r1, green: r2, blue: r3)

coefficient of x_3 is eliminated in row 2

↓

green plane adjusts so that

(plane normal) · $\vec{e}_3$ = 0



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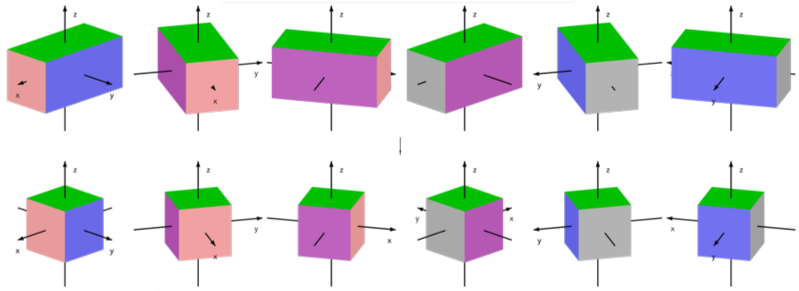
Step 7 of 7

Make pivot M[1,1] = 1

$$R1 \leftarrow \left(\frac{1}{M[1,1]}\right) \times R1$$

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Column space fills $\mathbb{R}^3$
Scaling space:
all columns scale
together along $\vec{e}_1$ direction by 0.5
scaling coefficient > 0
↓
directions are preserved



Row normal planes (red: r1, green: r2, blue: r3)
scaled plane remains unchanged