Column 1 of Q is calculated from column 1 of M

$$M = \begin{bmatrix} -0.34 & 1.21 & -2.41 \\ -2 & 0.38 & -1.62 \\ -0.89 & 0.88 & 0.03 \end{bmatrix}$$

by subtracting projections of $\vec{m}1$ on all
previously constructed non-zero orthonormal columns of Q

$$Q(\text{previous}) = \begin{bmatrix} * & * & * \\ * & * & * \\ * & * & * \end{bmatrix}$$

$$\vec{q}1 = \vec{m}1 = \begin{bmatrix} -0.34 \\ -2 \\ -0.89 \end{bmatrix}$$

$$\vec{q}1 \text{ normalized} = \frac{\vec{q}1}{\|\vec{q}1\|} = \begin{bmatrix} -0.153475802699923 \\ -0.902798839411309 \\ -0.401745483538033 \end{bmatrix}$$

$$Q(\text{updated}) = \begin{bmatrix} -0.153475802699923 & * & * \\ -0.902798839411309 & * & * \\ -0.401745483538033 & * & * \end{bmatrix}$$

Column 2 of Q is calculated from column 2 of M

$$M = \begin{bmatrix} -0.34 & 1.21 & -2.41 \\ -2 & 0.38 & -1.62 \\ -0.89 & 0.88 & 0.03 \end{bmatrix}$$

by subtracting projections of $\vec{m}2$ on all previously constructed non-zero orthonormal columns of Q

$$Q(\text{previous}) = \begin{bmatrix} -0.153475802699923 & * & * \\ -0.902798839411309 & * & * \\ -0.401745483538033 & * & * \end{bmatrix}$$

$$\vec{q}2 = \vec{m}2 - \left(\frac{\vec{m}2 \cdot \vec{q}1}{\vec{q}1 \cdot \vec{q}1} \right) \vec{q}1 = \begin{bmatrix} 1.07458748497259 \\ -0.416544206043564 \\ 0.525537828310614 \end{bmatrix}$$

$$\vec{q}2 \text{ normalized} = \frac{\vec{q}2}{\|\vec{q}2\|} = \begin{bmatrix} 0.848360417065187 \\ -0.328851416294149 \\ 0.414899203130488 \end{bmatrix}$$

$$Q(\text{updated}) = \begin{bmatrix} -0.153475802699923 & 0.848360417065187 & * \\ -0.902798839411309 & -0.328851416294149 & * \\ -0.401745483538033 & 0.414899203130488 & * \end{bmatrix}$$

Column 3 of Q is calculated from column 3 of M

$$M = \begin{bmatrix} -0.34 & 1.21 & -2.41 \\ -2 & 0.38 & -1.62 \\ -0.89 & 0.88 & 0.03 \end{bmatrix}$$

by subtracting projections of $\vec{m}3$ on all
previously constructed non-zero orthonormal columns of Q

$$Q(\text{previous}) = \begin{bmatrix} -0.153475802699923 & 0.848360417065187 & * \\ -0.902798839411309 & -0.328851416294149 & * \\ -0.401745483538033 & 0.414899203130488 & * \end{bmatrix}$$

$$\vec{q}3 = \vec{m}3 - \left(\frac{\vec{m}3 \cdot \vec{q}1}{\vec{q}1 \cdot \vec{q}1}\right)\vec{q}1 - \left(\frac{\vec{m}3 \cdot \vec{q}2}{\vec{q}2 \cdot \vec{q}2}\right)\vec{q}2 = \begin{bmatrix} -0.858619371698711 \\ -0.469649940476922 \\ 1.38340501947349 \end{bmatrix}$$

$$\vec{q}3 \text{ normalized} = \frac{\vec{q}3}{\|\vec{q}3\|} = \begin{bmatrix} -0.506685090310142 \\ -0.277147977728371 \\ 0.816369535013696 \end{bmatrix}$$

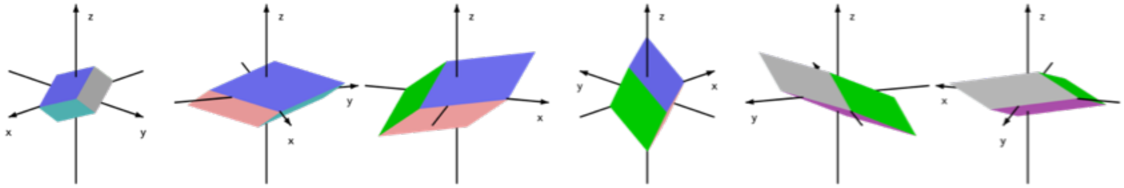
$$Q(\text{updated}) = \begin{bmatrix} -0.153475802699923 & 0.848360417065187 & -0.506685090310142 \\ -0.902798839411309 & -0.328851416294149 & -0.277147977728371 \\ -0.401745483538033 & 0.414899203130488 & 0.816369535013696 \end{bmatrix}$$

- We constructed Q by subtracting projections onto preceding vectors and normalizing
- Columns of Q are orthonormal, therefore: $Q^{-1} = Q^T$
- We compute $R = Q^T M$
- Decomposition $M = QR$ is shown

$$M = \begin{bmatrix} -0.153475802699923 & 0.848360417065187 & -0.506685090310142 \\ -0.902798839411309 & -0.328851416294149 & -0.277147977728371 \\ -0.401745483538033 & 0.414899203130488 & 0.816369535013696 \end{bmatrix} \begin{bmatrix} 2.21533293208944 & -0.882305305756673 & 1.82035843984699 \\ 0 & 1.26666386521193 & -1.49936233463666 \\ 0 & 0 & 1.69458187761781 \end{bmatrix}$$

Column directions during orthogonalization

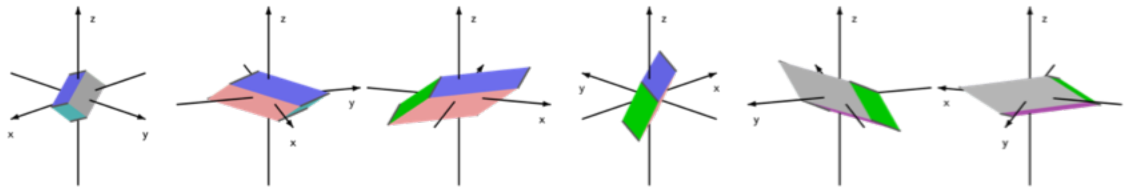
-0.34	1.21	-2.41
-2	0.38	-1.62
-0.89	0.88	0.03



column 1 normalized:
A2 = A1 × D1

$\vec{a}_1$ scaled so that $\hat{q}_1 = \vec{a}_1 / |\vec{a}_1|$

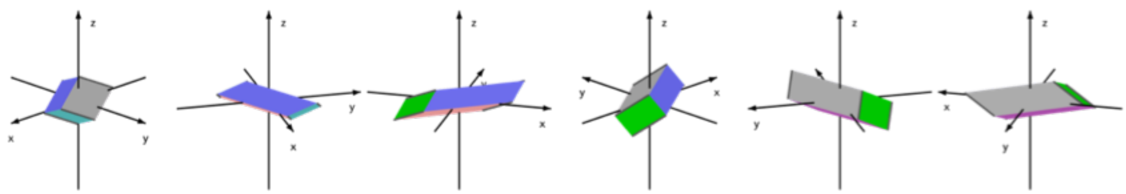
≈-0.153	1.21	-2.41
≈-0.903	0.38	-1.62
≈-0.402	0.88	0.03



column 2 orthogonalized:
A3 = A2 × S2

$\vec{a}_2$ orthogonalized so that $\vec{a}_2 \cdot \hat{q}_1 = 0$

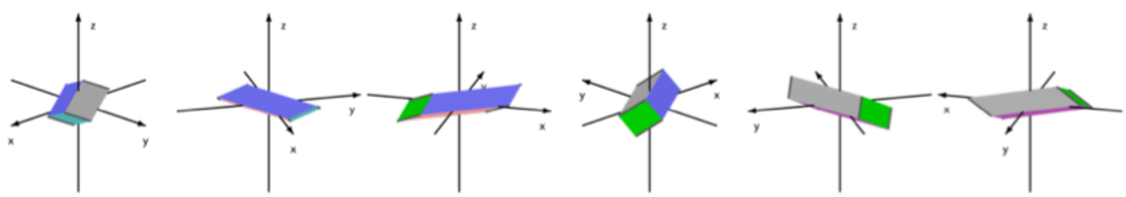
≈-0.153	≈1.075	-2.41
≈-0.903	≈-0.417	-1.62
≈-0.402	≈0.526	0.03



column 2 normalized:
A4 = A3 × D2

$\vec{a}_2$ scaled so that $\hat{q}_2 = \vec{a}_2 / |\vec{a}_2|$

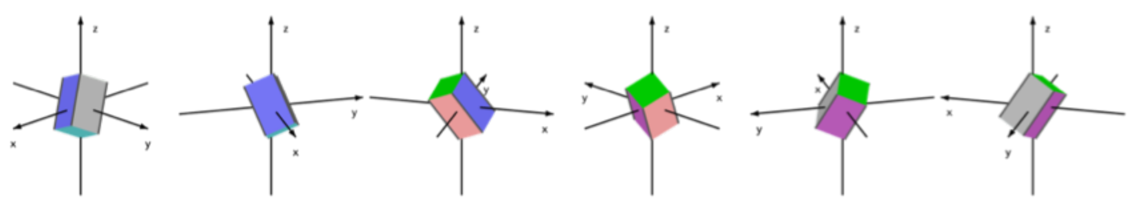
≈-0.153	≈0.848	-2.41
≈-0.903	≈-0.329	-1.62
≈-0.402	≈0.415	0.03



column 3 orthogonalized:
A5 = A4 × S3

$\vec{a}_3$ orthogonalized so that $\vec{a}_3 \cdot \hat{q}_1 = 0$, $\vec{a}_3 \cdot \hat{q}_2 = 0$

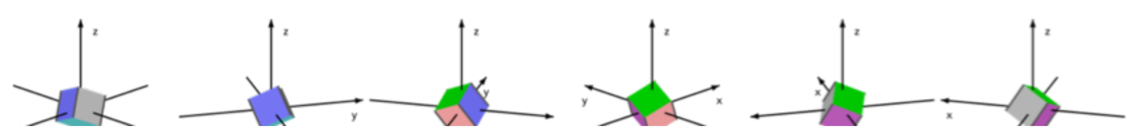
≈-0.153	≈0.848	≈-0.859
≈-0.903	≈-0.329	≈-0.47
≈-0.402	≈0.415	≈1.383

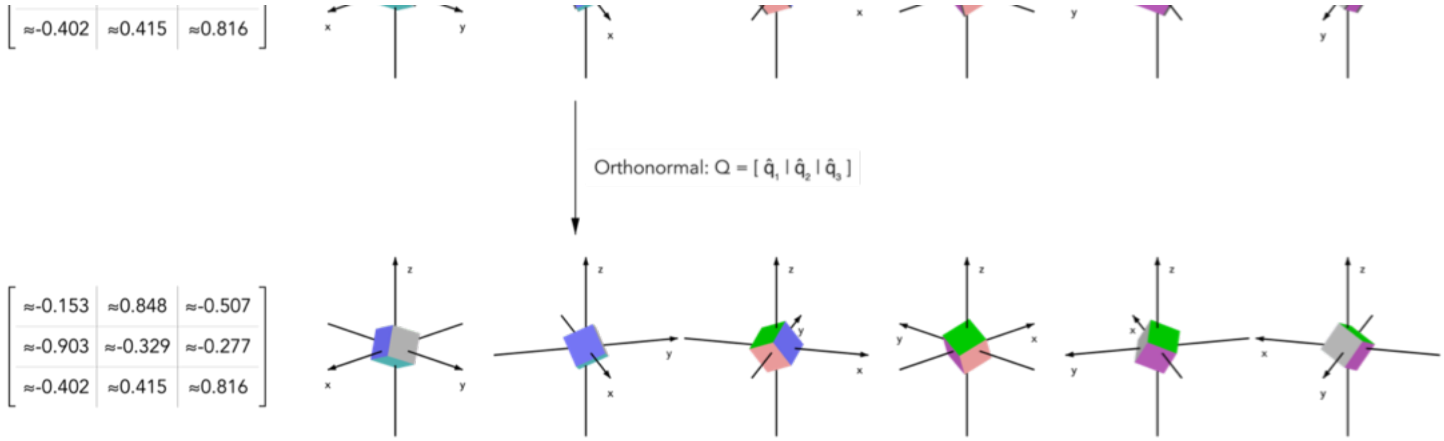


column 3 normalized:
A6 = A5 × D3

$\vec{a}_3$ scaled so that $\hat{q}_3 = \vec{a}_3 / |\vec{a}_3|$

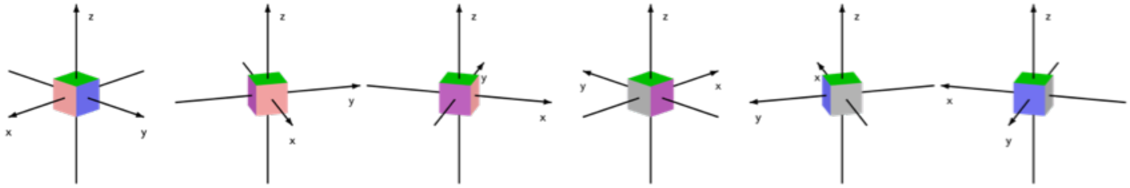
≈-0.153	≈0.848	≈-0.507
≈-0.903	≈-0.329	≈-0.277





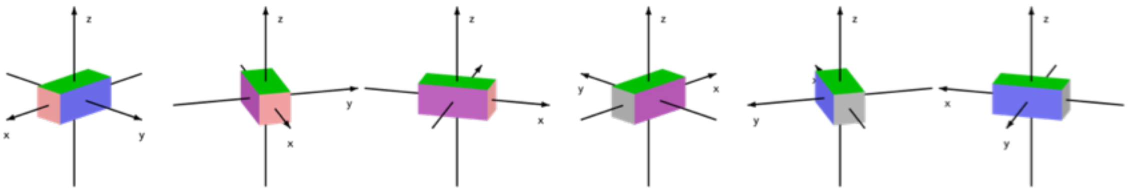
Stepwise construction of R during orthogonalization

1	0	0
0	1	0
0	0	1



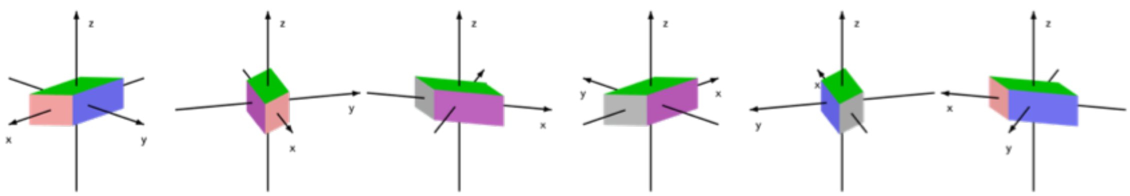
updated by inverse scaling:
$R2 = D1^{-1} \times R1 = D1^{-1}$

≈2.215	0	0
0	1	0
0	0	1



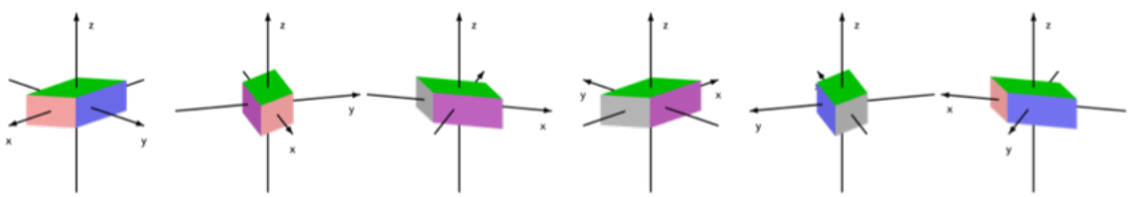
updated by inverse shear:
$R3 = S2^{-1} \times R2$

≈2.215	≈-0.882	0
0	1	0
0	0	1



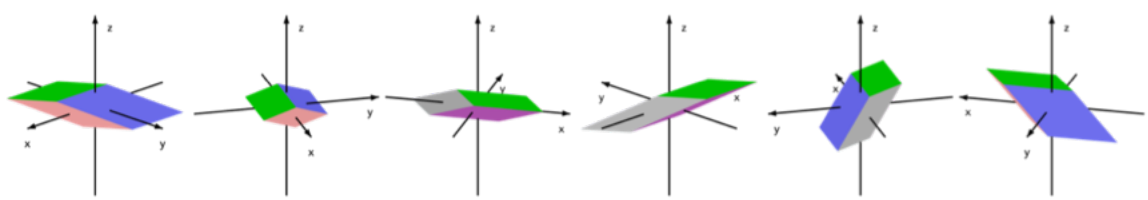
updated by inverse scaling:
$R4 = D2^{-1} \times R3$

≈2.215	≈-0.882	0
0	≈1.267	0
0	0	1



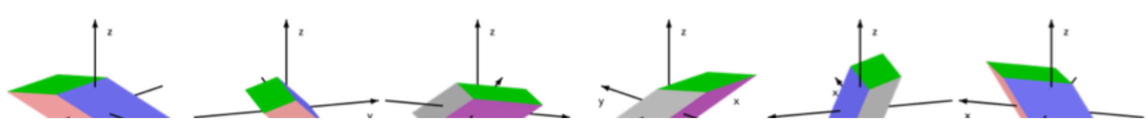
updated by inverse shear:
$R5 = S3^{-1} \times R4$

≈2.215	≈-0.882	≈1.82
0	≈1.267	≈-1.499
0	0	1

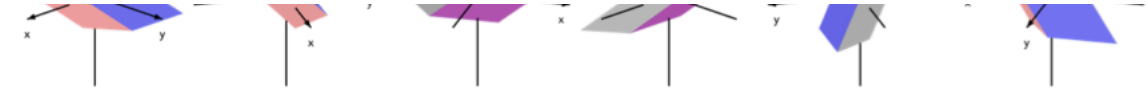


updated by inverse scaling:
$R6 = D3^{-1} \times R5$

≈2.215	≈-0.882	≈1.82
0	≈1.267	≈-1.499



0	0	≈1.695



≈2.215	≈-0.882	≈1.82
0	≈1.267	≈-1.499
0	0	≈1.695