Column 1 of Q is calculated from column 1 of M

$$M = \begin{bmatrix} 1 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

by subtracting projections of $\vec{m}1$ on all
previously constructed non-zero orthonormal columns of Q

$$Q(\text{previous}) = \begin{bmatrix} * & * \\ * & * \\ * & * \end{bmatrix}$$

$$\vec{q}1 = \vec{m}1 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

$$\vec{q}1 \text{ normalized} = \frac{\vec{q}1}{\|\vec{q}1\|} = \begin{bmatrix} 0.707106781186547 \\ 0.707106781186547 \\ 0 \end{bmatrix}$$

$$Q(\text{updated}) = \begin{bmatrix} 0.707106781186547 & * \\ 0.707106781186547 & * \\ 0 & * \end{bmatrix}$$

Column 2 of Q is calculated from column 2 of M

$$M = \begin{bmatrix} 1 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

by subtracting projections of $\vec{m}2$ on all previously constructed non-zero orthonormal columns of Q

$$Q(\text{previous}) = \begin{bmatrix} 0.707106781186547 & * \\ 0.707106781186547 & * \\ 0 & * \end{bmatrix}$$

$$\vec{q}2 = \vec{m}2 - \left(\frac{\vec{m}2 \cdot \vec{q}1}{\vec{q}1 \cdot \vec{q}1}\right)\vec{q}1 = \begin{bmatrix} 0.5 \\ -0.5 \\ 1 \end{bmatrix}$$

$$\vec{q}2 \text{ normalized} = \frac{\vec{q}2}{\|\vec{q}2\|} = \begin{bmatrix} 0.408248290463863 \\ -0.408248290463863 \\ 0.816496580927726 \end{bmatrix}$$

$$Q(\text{updated}) = \begin{bmatrix} 0.707106781186547 & 0.408248290463863 \\ 0.707106781186547 & -0.408248290463863 \\ 0 & 0.816496580927726 \end{bmatrix}$$

• We constructed Q by subtracting projections onto preceding vectors and normalizing
• Columns of Q are orthonormal, therefore: $Q^{-1} = Q^T$
• We compute $R = Q^T M$
• Decomposition M = QR is shown

$$M = \begin{bmatrix} 0.707106781186547 & 0.408248290463863 \\ 0.707106781186547 & -0.408248290463863 \\ 0 & 0.816496580927726 \end{bmatrix} \begin{bmatrix} 1.41421356237309 & 0.707106781186547 \\ 0 & 1.22474487139159 \end{bmatrix}$$
